Exercise 53

Use the definition of a derivative to find $f'(x)$ and $f''(x)$. Then graph f, f', and f'' on a common screen and check to see if your answers are reasonable.

$$f(x) = 3x^2 + 2x + 1$$

Solution

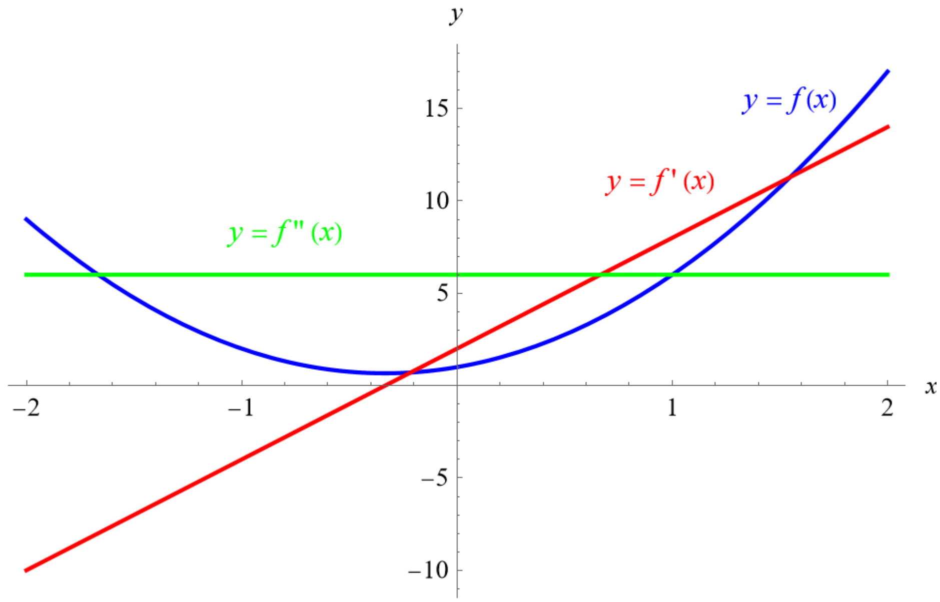
Use the definition of the derivative to find f'.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{[3(x+h)^2 + 2(x+h) + 1] - [3x^2 + 2x + 1]}{h} \\
&= \lim_{h\to 0} \frac{[3(x^2 + 2xh + h^2) + 2x + 2h + 1] - 3x^2 - 2x - 1}{h} \\
&= \lim_{h\to 0} \frac{(3x^2 + 6xh + 3h^2 + 2x + 2h + 1) - 3x^2 - 2x - 1}{h} \\
&= \lim_{h\to 0} \frac{6xh + 3h^2 + 2h}{h} \\
&= \lim_{h\to 0} (6x + 3h + 2) \\
&= 6x + 2
\end{aligned}$$

Use the definition of the derivative again to find f''.

$$\begin{aligned}
f''(x) &= \lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h} \\
&= \lim_{h\to 0} \frac{[6(x+h) + 2] - [6x + 2]}{h} \\
&= \lim_{h\to 0} \frac{(6x + 6h + 2) - 6x - 2}{h} \\
&= \lim_{h\to 0} \frac{6h}{h} \\
&= \lim_{h\to 0} 6 \\
&= 6
\end{aligned}$$

Below is a graph of $f(x)$, $f'(x)$, and $f''(x)$ versus x.



Notice that the red curve is negative where the blue curve is decreasing and positive where the blue curve is increasing. Since the red curve is increasing at a constant rate, the green curve is just a constant.